R16

Code No: 133BD

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, October - 2020

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 2 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

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1.a) State the necessary conditions for a function $f(z)$ to be analytic. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although the Cauchy-Riemann equations are satisfied.

b) Determine the analytic function $w = u + iv$, if $v = \log(x^2 + y^2) + x - 2y$. [8+7]

2.a) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0$.

b) If $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is an analytic function of z, then find $f(z)$ in terms of z. [7+8]

3.a) Find the Laurent series that represents the function $f(z) = z^2 \sin\frac{1}{z^2}$ in the domain $0 < |z| < \infty$.

b) Evaluate the integral $\int_C \frac{\cos \pi z}{z(z^2+1)} dz$ where C is the circle $|z| = 2$, described in the positive sense. [8+7]

4.a) Find the Taylor series expansion of the function $f(z) = \sin^3 z$ about $z = 0$.

b) Evaluate the integral $\int_C \frac{\cosh z}{(1-z)^{100}} dz$, where C is the closed circle $|z| = \frac{1}{2}$. [8+7]

5. Evaluate the real integral using contour integration $\int_0^\infty \frac{\cos ax}{1+x^2}dx$. [15]

6. Find the bilinear transformation which maps the points $(0, 1, \infty)$ into the points $(1, -z, -i)$. Find the image of the line $y = 5x$ under this transformation. [15]

7.a) Find the Fourier series of $f(x) = \begin{cases} 1 + x, & 0 < x < \pi \\ 1 - x, & -\pi < x < 0 \end{cases}$.

Deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

b) Express the function $f(x) = \begin{cases} 1 & for\ |x| \le 1, \\ 0 & for\ |x| > 1, \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x\ d\lambda}{\lambda}$. [8+7]

8. Write down the one dimensional heat equation. Find the temperature $u(x,t)$ in a slab whose ends $x = 0$ and $x = L$ are kept at zero temperature and whose initial temperature $f(x)$ is given by [15]

$$f(x) = \begin{cases} k, & when\ 0 < x < \frac{1}{2}L \\ 0, & when\ \frac{1}{2}L < x < L \end{cases}.$$

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